Grade Level/Course: Algebra 1

Lesson/Unit Plan Name: Connecting Arithmetic Sequences to Linear Equations

Rationale/Lesson Abstract:
This lesson is designed to strengthen students' understanding of arithmetic sequences by relating the content to their knowledge of linear equations.

This lesson is intended to be implemented after arithmetic sequences are introduced. Students should already be familiar with sequence notation, the common difference and the explicit formula: $a_n = a_1 + (n-1)d$.

Timeframe: 60 minutes

Common Core Standard(s):

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Instructional Resources/Materials:
Warm up (pg. 8)

Answers to Warm Up:

Write an equation in slope intercept form with a slope of -2 that goes through the point $(5,-4)$.

$$y = mx + b$$
$$-4 = -2(5) + b$$
$$-4 = -10 + b$$
$$-4 + 10 = -10 + b + 10$$
$$6 = b$$

$$y = -2x + 6$$

Find the common difference of the arithmetic sequence.

$$-\frac{1}{3}, -\frac{1}{8}, \frac{1}{12}, \frac{7}{24}, \ldots$$

$$d = a_2 - a_1$$
$$d = -\frac{1}{8} - \left(-\frac{1}{3}\right)$$
$$d = -\frac{1}{8}\bullet\frac{3}{3} + \frac{1}{3}\bullet\frac{8}{8}$$
$$d = -\frac{3}{24} + \frac{8}{24}$$
$$d = \frac{5}{24}$$

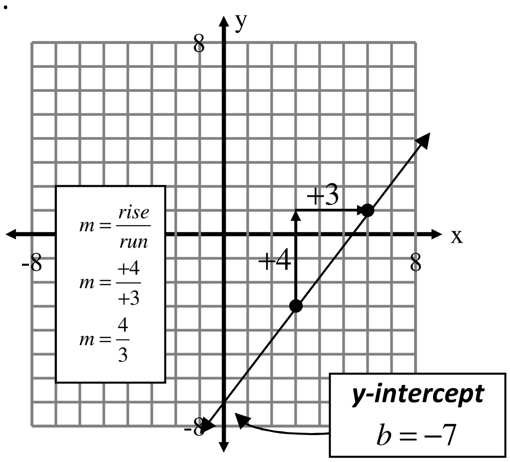
or

$$d = a_3 - a_2$$
$$d = \frac{1}{12} - \left(-\frac{1}{8}\right)$$
$$d = \frac{1}{12}\bullet\frac{2}{2} + \frac{1}{8}\bullet\frac{3}{3}$$
$$d = \frac{2}{24} + \frac{3}{24}$$
$$d = \frac{5}{24}$$

or

$$d = a_4 - a_3$$
$$d = \frac{7}{24} - \frac{1}{12}$$
$$d = \frac{7}{24} - \frac{1}{12}\bullet\frac{2}{2}$$
$$d = \frac{7}{24} - \frac{2}{24}$$
$$d = \frac{5}{24}$$

Write the equation of the line graphed below.



$$y = \frac{4}{3}x - 7$$

Find the slope of the line that goes through the points $(5,4)$ and $(2,-3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{(-3) - (4)}{(2) - (5)}$$
$$m = \frac{-7}{-3}$$
$$m = \frac{7}{3}$$

Activity/Lesson:
Example 1:

Write a rule for the n^{th} term of the arithmetic sequence:

$$-2, 1, 4, 7, \cdots$$

We can find the common difference and use the explicit formula:

$$d = a_2 - a_1$$
$$d = 1 - (-2)$$
$$d = 1 + 2$$
$$d = 3$$

$$a_n = a_1 + (n-1)d$$
$$a_n = -2 + (n-1)3$$
$$a_n = -2 + 3n - 3$$
$$a_n = 3n - 5$$

Point out that this equation looks similar to $y = mx + b$

Another way to find the y-intercept is to look at your table and go back to the "0^{th} term"…

0	−5
1	−2
2	1
3	4

-1 -3

If we think of each term as ordered pairs (n, a_n) *we can examine the sequence in a table:*

n	a_n
1	−2
2	1
3	4
4	7

Let $x = n$
$y = a_n$

x	y
1	−2
2	1
3	4
4	7

$+1$ $+3$

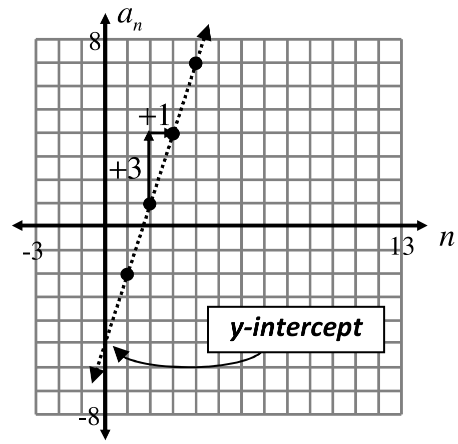
$$m = \frac{\Delta y}{\Delta x}$$
$$m = \frac{+3}{+1}$$
$$m = 3$$

$m = 3 \qquad (1, -2)$

$$y = mx + b$$
$$-2 = 3(1) + b$$
$$-2 = 3 + b$$
$$-2 - 3 = 3 + b - 3$$
$$-5 = b$$

$$y = 3x - 5$$
$$a_n = 3n - 5$$

We can graph the sequence to see the linear relationship:



$$m = \frac{rise}{run} \qquad b = -5$$
$$m = \frac{+3}{+1}$$
$$m = 3$$

$$y = 3x - 5$$
$$a_n = 3n - 5$$

You Try:

Write a rule for the n^{th} term of the arithmetic sequence:

$$-5, -6, -7, -8, \cdots$$

During the "you try", walk around the room and look for exemplary student. Hand select different students to explain their work to highlight the different methods as time permits

$d = a_2 - a_1$

$d = -6 - (-5)$

$d = -6 + 5$

$d = -1$

$a_n = a_1 + (n-1)d$

$a_n = -5 + (n-1)(-1)$

$a_n = -5 - n + 1$

$a_n = -n - 4$

If we think of each term as ordered pairs (n, a_n) we can examine the sequence in a table:

n	a_n
1	−5
2	−6
3	−7
4	−8

Let $x = n$, $y = a_n$

x	y
1	−5
2	−6
3	−7
4	−8

$+1$ (change in x), -1 (change in y)

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{-1}{+1}$$

$$m = -1$$

$m = -1 \qquad (1, -5)$

$y = mx + b$

$-5 = -1(1) + b$

$-5 = -1 + b$

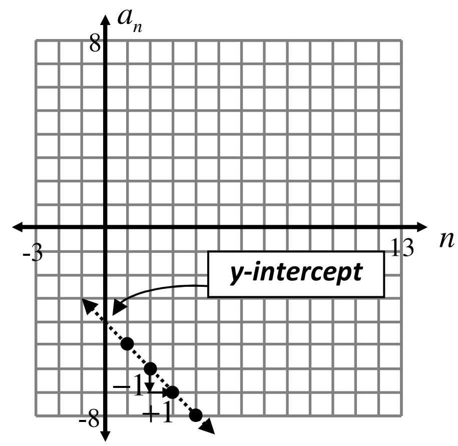
$-5 + 1 = -1 + b + 1$

$-4 = b$

$y = -x - 4$

$a_n = -n - 4$

We can graph the sequence to see the linear relationship:



$m = \frac{rise}{run}$ $\qquad b = -4$

$m = \frac{-1}{+1}$

$m = -1$

$y = -x - 4$

$a_n = -n - 4$

Example 2:

One term of an arithmetic sequence is $a_{11} = -52$ and the common difference is -4. Write an equation for the n^{th} term of the arithmetic sequence.

Using the Explicit Formula

$$a_n = a_1 + (n-1)d$$
$$a_{11} = a_1 + [(11)-1](-4)$$
$$-52 = a_1 + (10)(-4)$$
$$-52 = a_1 - 40$$
$$-52 + 40 = a_1 - 40 + 40$$
$$-12 = a_1$$

$$a_n = a_1 + d(n-1)$$
$$a_n = -12 + (-4)(n-1)$$
$$a_n = -12 - 4n + 4$$
$$a_n = -4n - 8$$

Using Slope-Intercept Form

$a_{11} = -52$ $\quad d = -4$

$(11, -52)$ $\quad m = -4$

$$y = mx + b$$
$$-52 = -4(11) + b$$
$$-52 = -44 + b$$
$$-52 + 44 = -44 + b + 44$$
$$-8 = b$$

$$y = -4x - 8$$
$$a_n = -4n - 8$$

Using Point-Slope Form

$a_{11} = -52$ $\quad d = -4$

$(11, -52)$ $\quad m = -4$

$$y - y_1 = m(x - x_1)$$
$$y - (-52) = -4(x - (11))$$
$$y + 52 = -4(x - 11)$$
$$y + 52 = -4x + 44$$
$$y + 52 - 52 = -4x + 44 - 52$$
$$y = -4x - 8$$
$$a_n = -4n - 8$$

You Try:

One term of an arithmetic sequence is $a_{26} = 30$ and the common difference is 5. Write an equation for the n^{th} term of the arithmetic sequence.

Using Sequence Notation

$$a_n = a_1 + (n-1)d$$
$$a_{26} = a_1 + ((26)-1)(5)$$
$$30 = a_1 + (25)(5)$$
$$30 = a_1 + 125$$
$$30 - 125 = a_1 + 125 - 125$$
$$-95 = a_1$$

$$a_n = a_1 + d(n-1)$$
$$a_n = -95 + 5(n-1)$$
$$a_n = -95 + 5n - 5$$
$$a_n = 5n - 100$$

Using Slope-Intercept Form

$a_{26} = 30$ $\quad d = 5$

$(26, 30)$ $\quad m = 5$

$$y = mx + b$$
$$30 = 5(26) + b$$
$$30 = 130 + b$$
$$30 - 130 = 130 + b - 130$$
$$-100 = b$$

$$y = 5x - 100$$
$$a_n = 5n - 100$$

Using Point-Slope Form

$a_{26} = 30$ $\quad d = 5$

$(26, 30)$ $\quad m = 5$

$$y - y_1 = m(x - x_1)$$
$$y - (30) = 5(x - (26))$$
$$y - 30 = 5(x - 26)$$
$$y - 30 = 5x - 130$$
$$y - 30 + 30 = 5x - 130 + 30$$
$$y = 5x - 100$$
$$a_n = 5n - 100$$

Example 3:

Write an equation for the n^{th} term of the arithmetic sequence. Then find a_{20}.

$$\frac{17}{2}, 10, \frac{23}{2}, 13, \cdots$$

$$d = a_2 - a_1$$
$$d = 10 - \frac{17}{2}$$
$$d = \frac{10}{1} \bullet \frac{2}{2} - \frac{17}{2}$$
$$d = \frac{20}{2} - \frac{17}{2}$$
$$d = \frac{3}{2}$$

$$a_n = a_1 + (n-1)d$$
$$a_n = \frac{17}{2} + (n-1)\frac{3}{2}$$
$$a_n = \frac{17}{2} + \frac{3}{2}n - \frac{3}{2}$$
$$a_n = \frac{3}{2}n + \frac{14}{2}$$
$$a_n = \frac{3}{2}n + 7$$

$$a_n = \frac{3}{2}n + 7$$
$$a_{20} = \frac{3}{2}(20) + 7$$
$$a_{20} = 30 + 7$$
$$a_{20} = 37$$

$a_2 = 10$ $\quad a_4 = 13$

$(2,10)$ $\quad (4,13)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{(13)-(10)}{(4)-(2)}$$
$$m = \frac{3}{2}$$

$$m = \frac{3}{2} \quad (2,10)$$
$$y = mx + b$$
$$10 = \frac{3}{2}(2) + b$$
$$10 = 3 + b$$
$$10 - 3 = 3 + b - 3$$
$$7 = b$$
$$y = \frac{3}{2}x + 7$$
$$a_n = \frac{3}{2}n + 7$$

$$a_n = \frac{3}{2}n + 7$$
$$a_{20} = \frac{3}{2}(20) + 7$$
$$a_{20} = 30 + 7$$
$$a_{20} = 37$$

You Try:

Write an equation for the n^{th} term of the arithmetic sequence. Then find a_{100}.

$$11, 8, 5, 2, \cdots$$

$$d = a_2 - a_1$$
$$d = 8 - 11$$
$$d = -3$$

$$a_n = a_1 + (n-1)d$$
$$a_n = 11 + (n-1)(-3)$$
$$a_n = 11 - 3n + 3$$
$$a_n = -3n + 14$$

$$a_n = -3n + 14$$
$$a_{100} = -3(100) + 14$$
$$a_{100} = -300 + 14$$
$$a_{100} = -286$$

$a_1 = 11 \qquad a_2 = 8$

$(1, 11) \qquad (2, 8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{(8) - (11)}{(2) - (1)}$$
$$m = \frac{-3}{1}$$
$$m = -3$$

$m = -3 \quad (1, 11)$

$$y - y_1 = m(x - x_1)$$
$$y - 11 = -3(x - 1)$$
$$y - 11 = -3x + 3$$
$$y - 11 + 11 = -3x + 3 + 11$$
$$y = -3x + 14$$
$$a_n = -3n + 14$$

$$a_n = -3n + 14$$
$$a_{100} = -3(100) + 14$$
$$a_{100} = -300 + 14$$
$$a_{100} = -286$$

Warm-Up

Algebra: F.BF.1

Write an equation in slope intercept form with a slope of -2 that goes through the point $(5,-4)$.

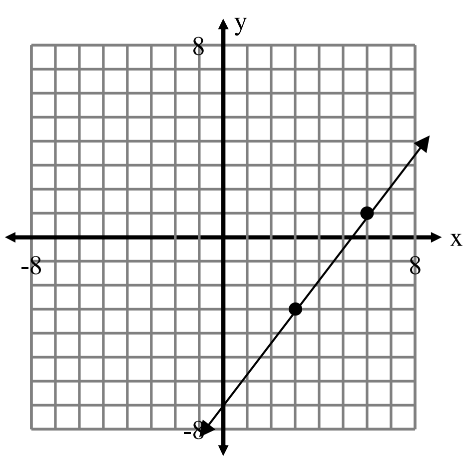
Algebra: F.BF.2

Find the common difference of the arithmetic sequence.

$$-\frac{1}{3},-\frac{1}{8},\frac{1}{12},\frac{7}{24},\ldots$$

Algebra: F.IF.4

Write the equation of the line graphed below.



Algebra: F.IF.6

Find the slope of the line that goes through the points $(5,4)$ and $(2,-3)$.